Higher Order Functions

September 25, 2018

Warm-up: filter out even numbers

Using filter, write a function that returns all odd numbers from a list of numbers.

Recap

First-class functions: functions that are treated just like other values in the language, including being able to appear in all syntactic environments.

Higher-order functions: functions that take functions as arguments.

Properties of Map

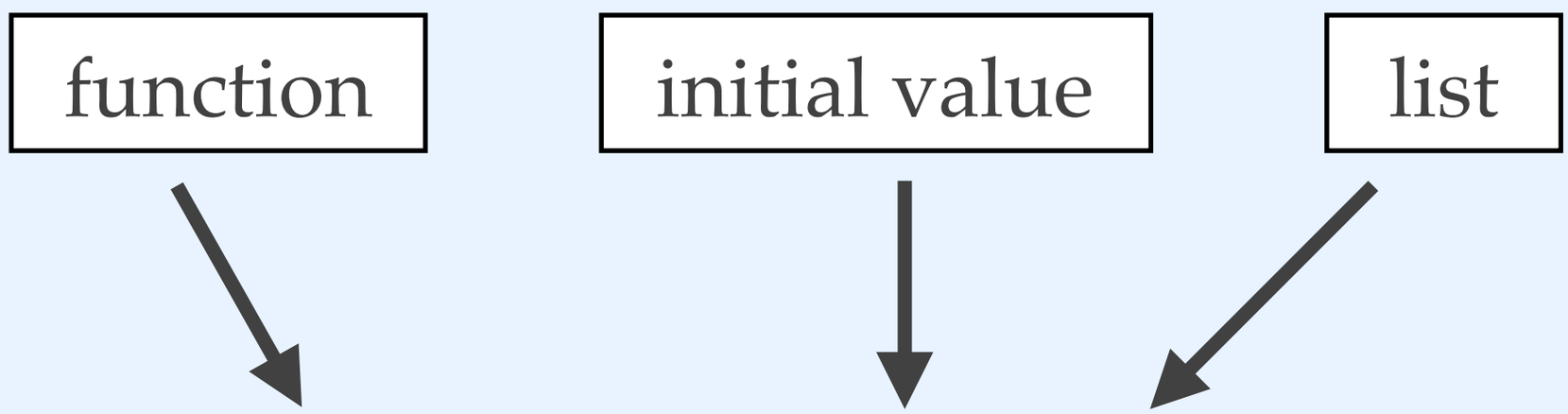
- Input items and return items do not need to be of the same type
- Preserves the length of the original list

Properties of Filter

- Function given as argument must return a boolean
- Does not preserve the length of list
- Returns copies of items from the original list

Fold: returning a single value

Fold is a higher-order function that takes a list and returns a single value. It is also known as reduce.




```
> (fold (lambda (x,y) (+ x y)) 0 (list 1 2 3) )

   6
```

Fold: returning a single value

```
(define (add x y) (+ x y))
```

```
(fold add 0 (list 1 2 3) )
```

```
(fold add (+ 1 0) (list 2 3))
```

```
(fold add (+ 2 1) (list 3))
```

```
(fold add (+ 3 3) (list ))
```

Foldl and Foldr

```
(define (add x y) (+ x y))
```

```
(foldl add 0 (list 1 2 3))
(foldl add (+ 1 0) (list 2 3))
(foldl add (+ 2 1) (list 3))
(foldl add (+ 3 3) (list ))
```

```
(foldr add 0 (list 1 2 3))
(foldr add (+ 3 0) (list 2 3))
(foldr add (+ 2 3) (list 3))
(foldr add (+ 1 5) (list ))
```

Properties of Fold

- Returns a single value of any type
- Takes an initial value as an argument, as well as the list and the function to apply
- Function supplied must have two arguments

Fold's initial value argument

- What return type do you want?
- What initial value do you need?

Exercise: list and

Write a version of and that takes a list.

Return true if all items in the list are true and false otherwise.

Use one of the built-in higher-order functions that we have discussed.

Exercise: list xor

Write a function that returns true if and only if 1 item in the list is true.

Use one of the built-in higher-order functions that we have discussed.

Properties of Map and Fold

One property of map is that mapping function f over list l, and then mapping function g over the result, is equivalent to mapping the composition of f and g over l.

```
(define (add-5 x) (+ x 5))
(define (multiply-by-10 x) (* x 10))
(define numbers (list 1 2 3))
```

```
> (map multiply-by-10
       (map add-5 numbers))

(60 70 80)
```

```
> (map (lambda (x)
              (multiply-by-10 (add-5 x)))
            numbers
(60 70 80)
```

Properties of Map and Fold

Similarly, mapping function f over list l and then folding function g over the result is equivalent to folding the composition of f and g over l.

```
(define (add-5 x) (+ x 5))
(define (sum x y) (+ x y))
(define numbers (list 1 2 3))
```

```
>(fold sum
       0
       (map add-5 numbers))
21
```

```
> (fold (lambda (x y)
                (+ (add-5 x) y))
              numbers
21
```